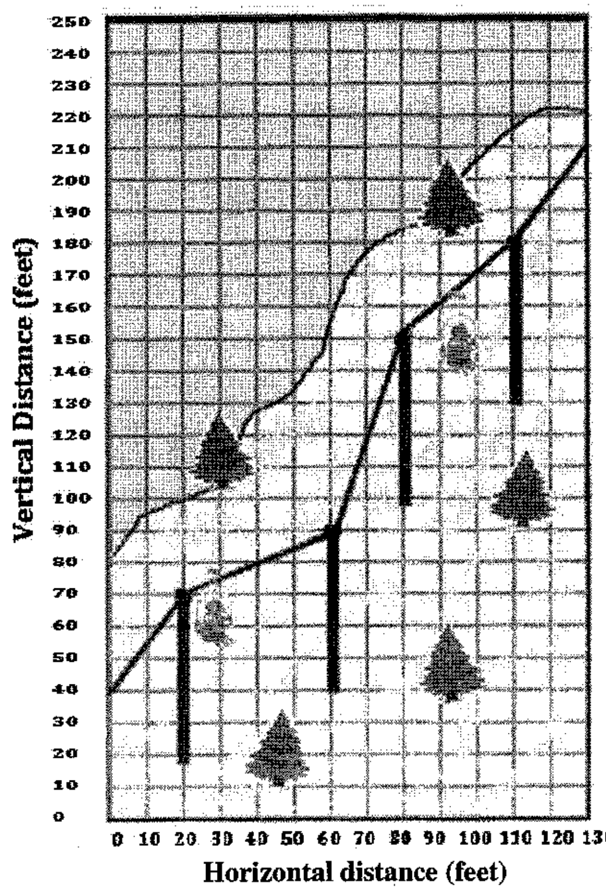
5-1 Rate of Change and Slope

The diagram at the right shows a chair lift up a mountain.



1. What is the vertical and horizontal change for each section of the chair lift?

   Section 1: vertical Δ 2 horizontal Δ 4

   Section 2: vertical Δ 6 horizontal Δ 2

   Section 3: vertical Δ 3 horizontal Δ 3

2. Now, find the ratio of the vertical change to the horizontal change for each section.

   Ratio for section 1: $\frac{2}{4} = \frac{1}{2}$ or 1:2

   Ratio for section 2: $\frac{6}{2} = \frac{3}{1}$ or 3:1

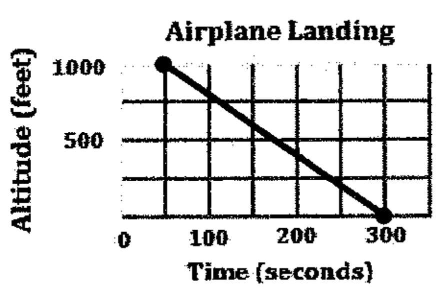
   Ratio for section 3: $\frac{3}{3} = \frac{1}{1}$ or 1:1

3. Now put the ratios in order from least steep to most steep. What do you notice?

   $\frac{1}{2}, \frac{1}{1}, \frac{3}{1}$ The larger the ratio, the steeper the line.

Example 1: Finding Rate of Change Using a Graph

The graph shows the altitude of an airplane as it comes in for a landing. Find the rate of change.



Step 1: Find the vertical change.

Step 2: Find the horizontal change.

Step 3: Make a ratio. $\frac{\text{vert}\Delta}{\text{hort.}\Delta}$

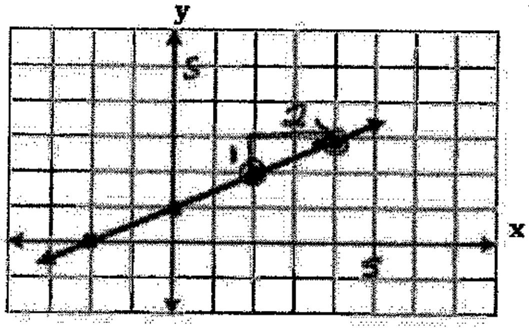
Step 4: Reduce

$\frac{\text{vert}\,\Delta}{\text{hort}\,\Delta} = \frac{-1000}{250} = -4$ or $-4:1$

Explain what this rate of change means.

This means the plane is descending at 4 feet per second.

Example 2: Finding Slope Using a Graph



Step 1: Locate 2 points on the line (2,2) and (4,3)

Step 2: Find the vertical and horizontal change.

Step 3: Make a ratio and reduce if needed.

$$\frac{\text{vert. }\Delta}{\text{horiz. }\Delta} = \frac{1}{2} \rightarrow \boxed{m = \frac{1}{2}}$$

A simple way to look at slopes:

* I have found this method to be more successful than slope triangles.

1.  $m = 1$

2.  $m = 2$

3.  $m = 3$

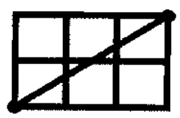
4.  $m = -1$

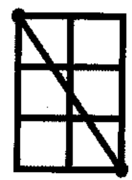
5.  $m = -2$

6.  $m = -3$

7.  $m = \frac{1}{2}$

8.  $m = \frac{1}{3}$

9.  $m = \frac{2}{3}$

10.  $m = \frac{-3}{2}$

√ Understanding Check:

Give the slope of each line

1.  $m = 2$

2.  $m = -2$

3.  $m = \frac{1}{2}$

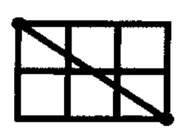
4.  $m = -\frac{1}{2}$

5.  $m = 3$

6.  $m = -\frac{1}{3}$

7.  $m = -4$

8.  $m = \frac{1}{4}$

9.  $m = -\frac{2}{3}$

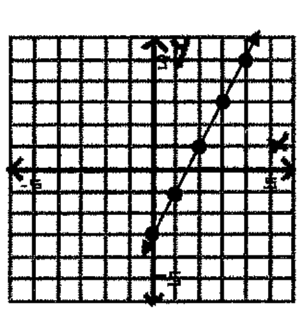
10.  $m = 1$

√ Understanding Check:

Find the slope of each line.

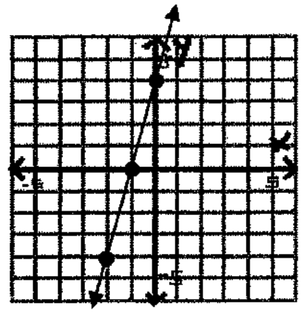
* I use highlighters to help students see the boxes that are between the points.

1.



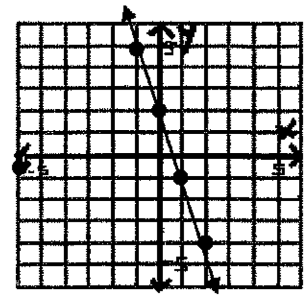
m = 2

2.



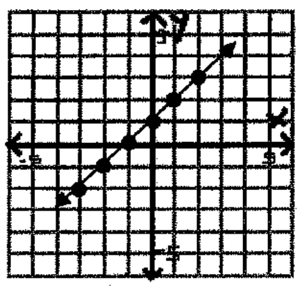
m = 4

3.



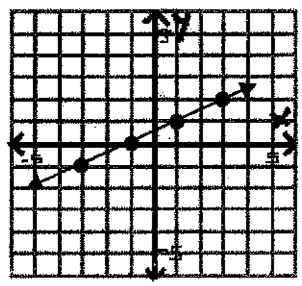
m = -3

4.



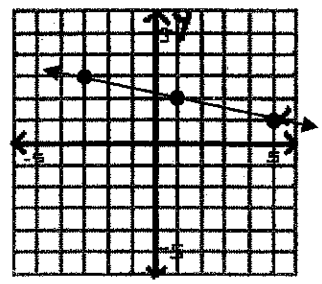
m = 1

5.



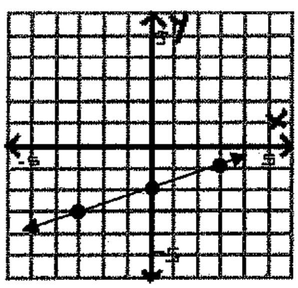
m = $\frac{1}{2}$

6.



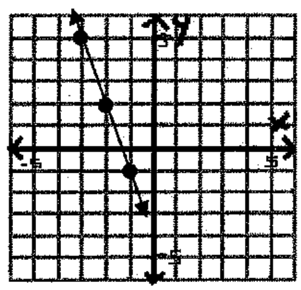
m = $\frac{-1}{4}$

7.



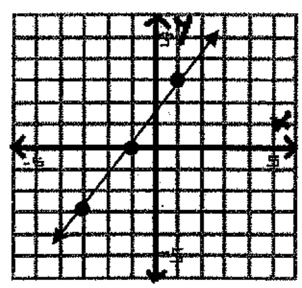
m = $\frac{1}{3}$

8.



m = -3

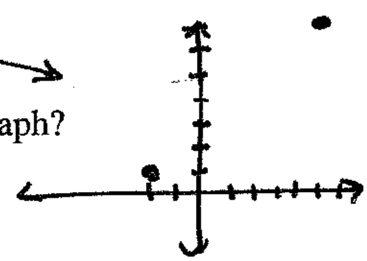
9.



m = $\frac{3}{2}$

Example 3: Finding Slope Using Points

Can you find the slope two points without seeing the graph?



* we draw this anyway. ☺

Find the slope of the line passing through (-2, 1) and (6, 7).

$x_1\ y_1$ $\quad$ $x_2\ y_2$

Step 1: Subtract the y's → $y_2 - y_1$

Step 2: Subtract the x's → $x_2 - x_1$

Step 3: Make a ratio

Step 3: Reduce if needed.

$$\frac{\text{vertical } \Delta}{\text{horizontal } \Delta} = \frac{7-1}{6-(-2)} = \frac{6}{8} = \frac{3}{4}$$

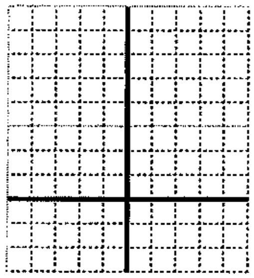
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ ← Slope formula!

Slope formula

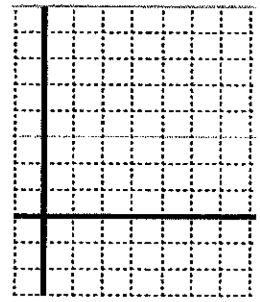
$$\text{Slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = m$$

Finding Slope From 2 Points Practice Activity

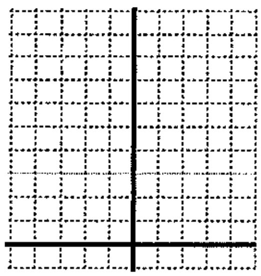
1. (,) and (,)



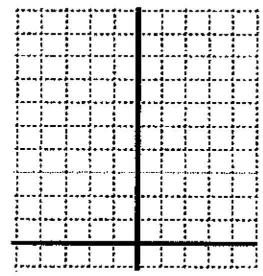
2. (,) and (,)



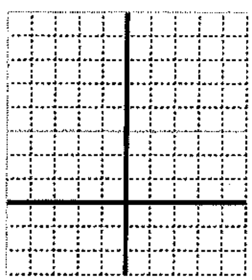
3. (,) and (,)



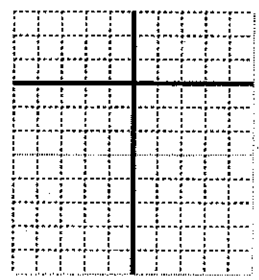
4. (,) and (,)



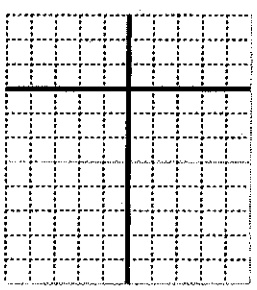
5. (,) and (,)



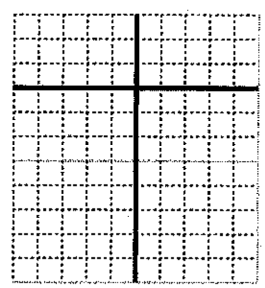
6. (,) and (,)



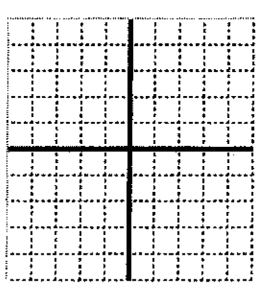
7. (,) and (,)



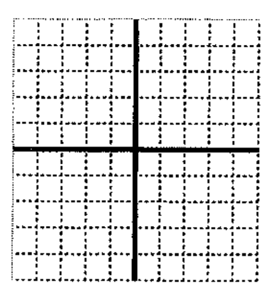
8. (,) and (,)



9. (,) and (,)



10. (,) and (,)

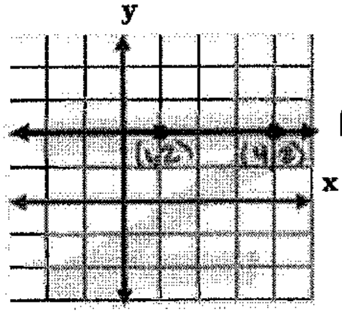


Example 4: Horizontal and Vertical Lines

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Find the slope of each line.

a.

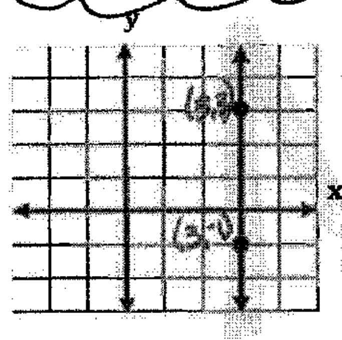


$m = \dfrac{2-2}{4-1}$

$m = \dfrac{0}{3}$

$m = 0$

b.



$m = \dfrac{3 - -1}{3 - 3}$

$m = \dfrac{4}{0}$

m = undefined

or No Slope

Find the slope of the line passing through each set of points:

a. (3, 4) and (-3, 4) b. (2, 3) and (2, -3) c. (5, -1) and (-3, -1)

$m = \dfrac{4-4}{-3-3} = \dfrac{0}{-6} = 0$

$m = 0$

* Same y-values

$m = \dfrac{-3-3}{2-2} = \dfrac{-6}{0}$

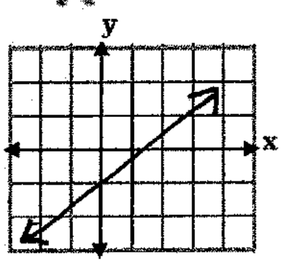
m = und.

* Same x-values

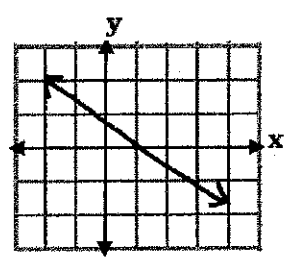
$m = \dfrac{-1 - -1}{-3 - 5} = \dfrac{0}{-8} = 0$

$m = 0$

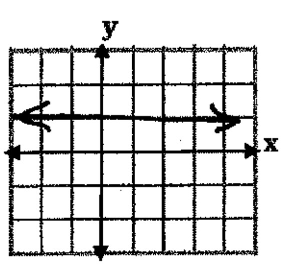
Slopes of Lines Summary



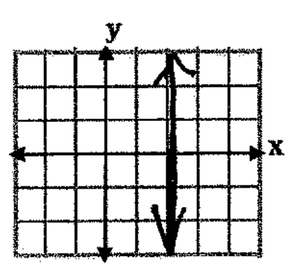
A line with positive slope slants upward from from left to right.



A line with negative slope slants downward from from left to right.



A line wth a slope of Ø is horizontal



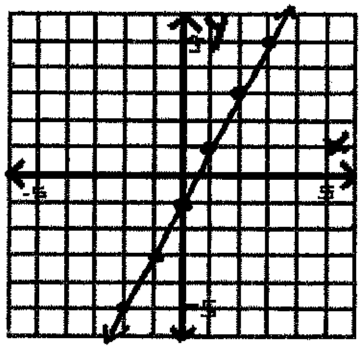
A line wth a slope of und. is vertical

5-2 Slope-Intercept Form

Graph the following linear equations by making a table of values for each:

a. $y = 2x - 1$

x	y
-2	-5
-1	-3
0	-1
1	1
2	3

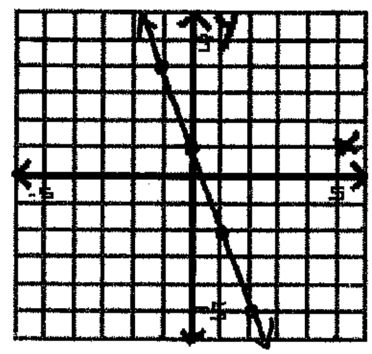


What is the slope of this line? 2

Where does this graph cross the y-axis? (0, -1)

b. $y = -3x + 1$

x	y
-2	7
-1	4
0	1
1	-2
2	-5



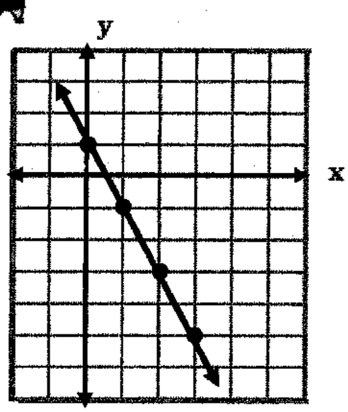
What is the slope of this line? -3

Where does this graph cross the y-axis? (0, 1)

Compare the slope of each line to its equation. Do you notice any patterns?
The slope is the same as the coefficient of x.

Compare the point where each graph crosses the y-axis to its equation. Do you notice any patterns? The y-intercept is the same as the constant in the rule.

Vocabulary:



The slope of a line is its rate of change from left to right and is referred to as m.

The slope of the given graph is -2.

The y-intercept of a line is the point at which the line crosses the y-axis and is referred to as b.

The y-intercept of the given graph is (0, 1).

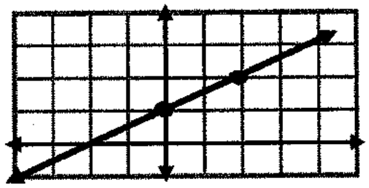
The slope intercept form of a linear equation is: $y = mx + b$

So, the slope intercept form of the line above is: $y = -2x + 1$

slope y-int.

Example 1: Writing an Equation from a Graph

Find the equation of the line using points (0, 1) and (2, 2).



Step 1: Find the slope (m)
Step 2: Find the y-intercept (b)
Step 3: write the y = mx+b form of the line.

$m = \frac{1}{2}$ $b = 1$

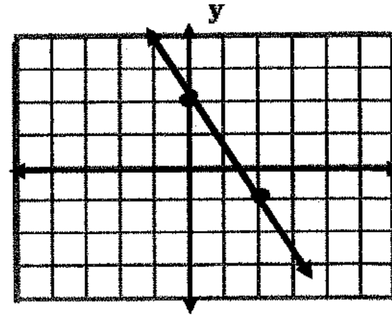
$y = \frac{1}{2}x + 1$

√ **Understanding Check:**

Which equation models the linear function shown in the graph?

A. $y = -\frac{2}{3}x + 2$ B. $y = -\frac{3}{2}x + 2$

C. $y = 2x - \frac{3}{2}$ D. $y = 2x - \frac{2}{3}$



$m = -\frac{3}{2}$ $b = 2$

Example 2: Identifying the Slope and y–intercept

What are the slope and y–intercept of y = 3x – 5 ?

slope (m) = 3
y-int (b) = −5 or (0,−5)

What are the slope and y–intercept of y = -4x + 6 ?

Slope (m) = −4
y-int (b) = 6 or (0,6)

√ **Understanding Check:**

Find the slope and y–intercept of each of the following lines:

a. y = –2x + 5
m = −2
b = (0,5)

b. y = 4x – 6
m = 4
b = (0,−6)

c. y = x + 2
m = 1
b = (0,2)

d. y = -x – 4
m = −1
b = (0,−4)

e. y = 2x – 5
m = 2
b = (0,−5)

f. $y = \frac{7}{6}x - \frac{3}{4}$
m = $\frac{7}{6}$
b = $(0, \frac{-3}{4})$

g. y = 3x
m = 3
b = (0,0)

h. y = 4
m = Ø
b = (0,4)

Example 3: Writing an Equation

Write an equation of the line with slope $=\frac{3}{8}$ and y–intercept 6.

$y = \frac{3}{8}x + 6$

√ Understanding Check:

Write an equation of the line with:

a. slope $=\frac{2}{5}$ and y–intercept (0,–1)

$y = \frac{2}{5}x - 1$

b. slope = -2 and y-intercept (0, 4)

$y = -2x + 4$

c. slope = 6 and y-intercept (0, -8)

$y = 6x - 8$

d. slope = -1 and y-intercept (0, 2)

$y = -x + 2$

Example 4: Graphing Equations from the Slope–Intercept Form of the Line

Step 1: Locate and plot the y-intercept.

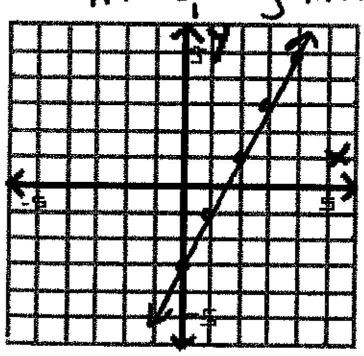
Step 2: Write the slope as a ratio.

Step 3: Follow / count the slope to the right!

Step 4: Plot points to the right, then left.

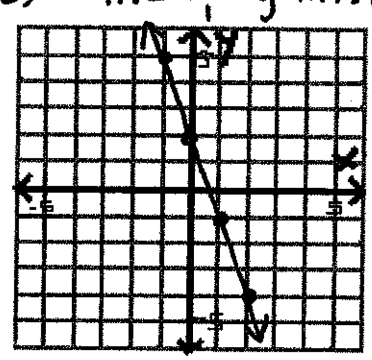
a. Graph y = 2x – 3

$m = \frac{2}{1}$ y-int: (0,-3)



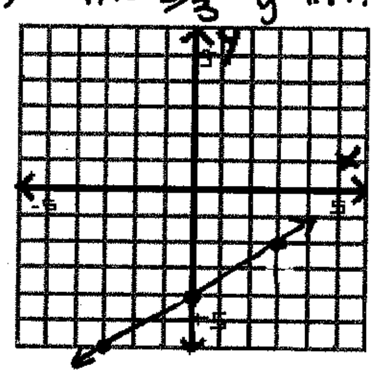
b. Graph y = -3x + 2

$m = \frac{-3}{1}$ y-int: (0,2)



c. Graph $y = \frac{2}{3}x - 4$

$m = \frac{2}{3}$ y-int: (0,-4)

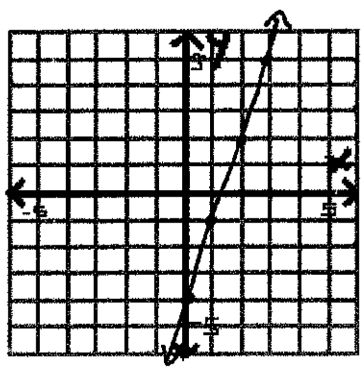


Important Reminder! The slope always begins to the right.

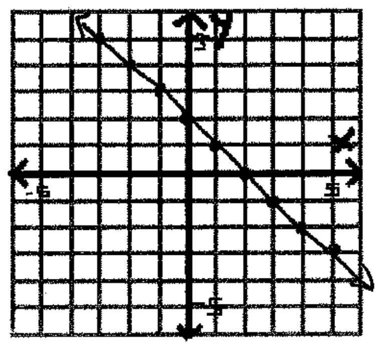
√ Understanding Check:

Graph each equation using the slope and y–intercept. (Show all steps on each graph)

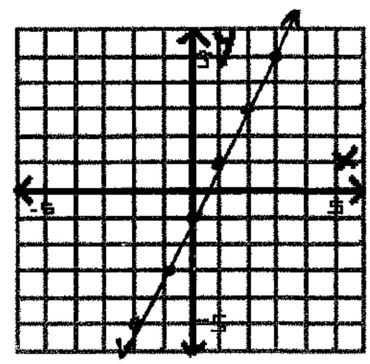
a. $y = 3x - 4$



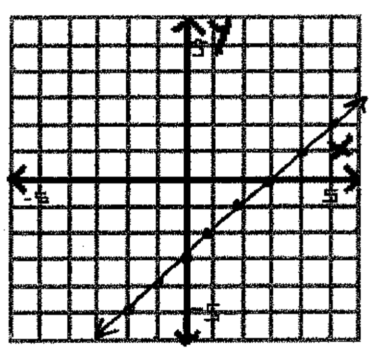
b. $y = -x + 2$



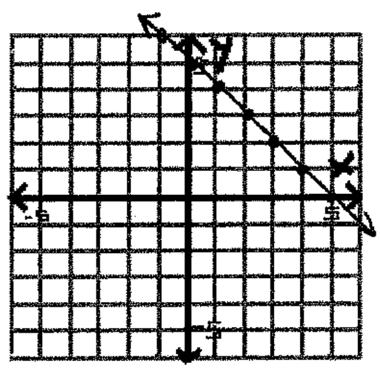
c. $y = 2x - 1$



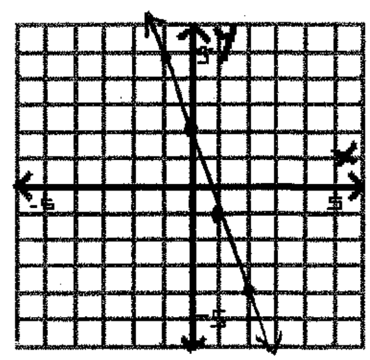
d. $y = x - 3$



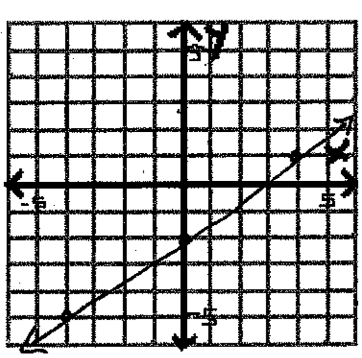
e. $y = -x + 5$



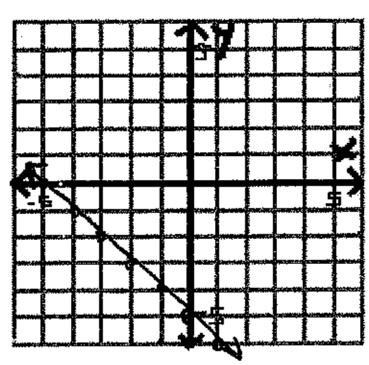
f. $y = -3x + 2$



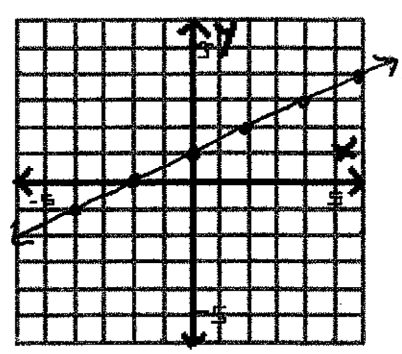
g. $y = \frac{3}{4}x - 2$



h. $y = -x - 5$



i. $y = \frac{1}{2}x + 1$



Transforming Equations into Slope-Intercept Form (y = mx + b):

The goal is to get $\underline{\ y\ }$ by itself and rest in $\underline{\ mx+b\ }$ form. Then name the slope and y-intercept of each line:

Easy Level:

$2x + y = 5$

$-2x$

$\boxed{y = -2x + 5}$

m = -2

y-int = $(0, 5)$

Medium Level:

$3y + 6x = -9$

$\frac{\cancel{3}y}{\cancel{3}} = \frac{-6x}{3} - \frac{9}{3}$

$\boxed{y = -2x - 3}$

m = -2

y-int = $(0, -3)$

Hard Level:

$2x - 4y = 3$

$-2x$

$\frac{-\cancel{4}y}{-\cancel{4}} = \frac{-2x}{-4} + \frac{3}{-4}$

$\boxed{y = \frac{1}{2}x - \frac{3}{4}}$

m = $\frac{1}{2}$

y-int = $(0, -3/4)$

√ Understanding Check:

Change each of the standard form equations back into slope-intercept form (y=mx+b). Then name the slope and y-intercept.

Solving for y – Easy Level

1. $14 + y = 8x$

-14

$y = 8x - 14$

m = 8

y-int = $(0, -14)$

2. $y - 9 = 6x$

$+9$

$y = 6x + 9$

m = 6

y-int = $(0, 9)$

3. $y - 8x = 16$

$+8x$

$y = 8x + 16$

m = 8

y-int = $(0, 16)$

4. $10 = y - 7x$

$+7x$

$y = 7x + 10$

m = 7

y-int = $(0, 10)$

5. $-8 + y = -2x$

$+8$

$y = -2x + 8$

m = -2

y-int = $(0, 8)$

6. $-6 = y + 8x$

$-8x$

$y = -8x - 6$

m = -8

y-int = $(0, -6)$

7. $-10 + y = 4x$

$+10$

$y = 4x + 10$

m = 4

y-int = $(0, 10)$

8. $y - 15x = -5$

$+15x$

$y = 15x - 5$

m = 15

y-int = $(0, -5)$

Solving for y – Medium Level

1. $14 + 2y = 8x$

-14

$\frac{\cancel{2}y}{\cancel{2}} = \frac{8x}{2} - \frac{14}{2}$

$y = 4x - 7$

m = 4
y-int = (0, -7)

2. $-3y - 9 = 6x$

$+9$

$\frac{-\cancel{3}y}{-\cancel{3}} = \frac{6x}{-3} + \frac{9}{-3}$

$y = -2x - 3$

m = -2
y-int = (0, -3)

3. $4y - 8x = 16$

$+8x$

$\frac{\cancel{4}y}{\cancel{4}} = \frac{8x}{4} + \frac{16}{4}$

$y = 2x + 4$

m = 2
y-int = (0, 4)

4. $10 = -y - 7x$

$+7x$

$\frac{\cancel{-}y}{\cancel{-1}} = \frac{7x}{-1} + \frac{10}{-1}$

$y = -7x - 10$

m = -7
y-int = (0, -10)

Solving for y – Hard Level

1. $x + 2y = 8$

$-x$

$\frac{\cancel{2}y}{\cancel{2}} = \frac{-x}{2} + \frac{8}{2}$

$y = -\frac{1}{2}x + 4$

m = $-\frac{1}{2}$
y-int = (0, 4)

2. $-3y - 6 = 2x$

$+6$

$\frac{-\cancel{3}y}{-\cancel{3}} = \frac{2x}{-3} + \frac{6}{-3}$

$y = -\frac{2}{3}x - 2$

m = $-\frac{2}{3}$
y-int = (0, -2)

3. $10 = -5y - 2x$

$+2x$

$\frac{-\cancel{5}y}{-\cancel{5}} = \frac{2x}{-5} + \frac{10}{-5}$

$y = -\frac{2}{5}x - 2$

m = $-\frac{2}{5}$
y-int = (0, -2)

4. $4y - 3x = 1$

$+3x$

$\frac{\cancel{4}y}{\cancel{4}} = \frac{3x}{4} + \frac{1}{4}$

$y = \frac{3}{4}x + \frac{1}{4}$

m = $\frac{3}{4}$
y-int = $(0, \frac{1}{4})$

√ Understanding Check:

Solve each equation for y, then graph using the y = mx + b shortcut.

1. $4x + y = 3$

$-4x$

$\boxed{y = -4x + 3}$

2. $-6x + 2y = -8$

$+6x$

$\frac{\cancel{2}y}{\cancel{2}} = \frac{6x}{2} - \frac{8}{2}$

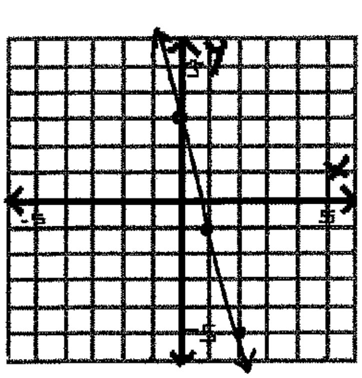
$\boxed{y = 3x - 4}$

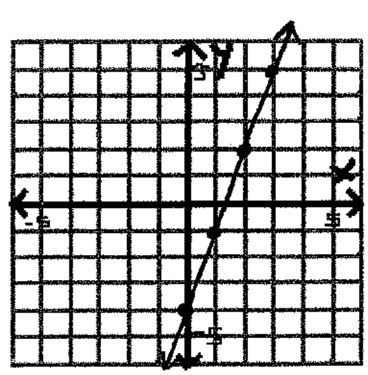
3. $2x - 6y = -6$

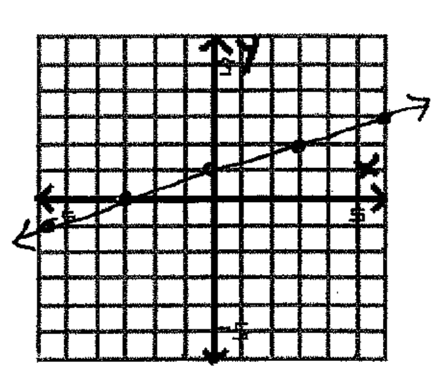
$-2x$

$\frac{-\cancel{6}y}{-\cancel{6}} = \frac{-2x}{-6} - \frac{6}{-6}$

$y = \frac{1}{3}x + 1$







5–3 Standard Form

Vocabulary:

The standard form of a linear equation is where A, B, and C are real numbers and A and B are not zero.

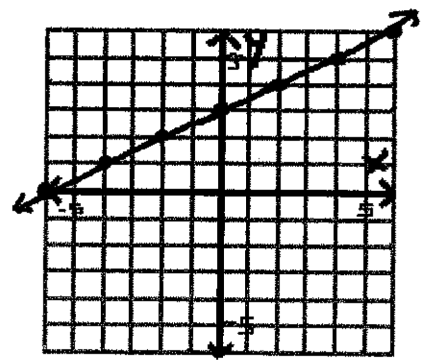
$Ax + By = C$ Example: $2x + 3y = 6$

Investigating Standard Form:

1. Graph the equation $-2x + 4y = 12$ by making a table of values for the domain $\{-6, -4, -2, 0, 2, 4, 6\}$.

*I usually use a blank sheet of paper to show scratch work to help students think.

x	$-2x + 4y = 12$	y
-6	$-2(-6) + 4y = 12$	0
-4	$-2(-4) + 4y = 12$	1
-2	$-2(-2) + 4y = 12$	2
0	$-2(0) + 4y = 12$	3
2	$-2(2) + 4y = 12$	4
4	$-2(4) + 4y = 12$	5
6	$-2(6) + 4y = 12$	6



2. What is the y–intercept on the graph? (0,3)

3. What is the x–intercept on the graph? (-6,0)

4. Check the table above, are the x and y intercept on the table? yes

What do all y-intercepts have? zero for the x-value

What do all x-intercepts have? zero for the y-value

5. Try substituting y = 0 into the original equation. What do you get for x?

$$-2x + 4(0) = 12$$
$$-2x = 12$$
$$x = -6$$

6. Explain how you could use algebra to find where the line will intercept both axis without having to make a table.

First, put ∅ in for x, and solve for y.
Then, put ∅ in for y, and solve for x.

Example 1: Finding x– and y–intercepts

Find the x– and y–intercepts of : $-2x + 4y = 8$

Step 1: substitute ø in for x and solve for y. → $\boxed{-2(0)+4y=8}$ →

Step 2: substitute ø in for y and solve for x. → $\boxed{-2x+4(0)=8}$ →

x	y
0	2
-4	0

√ Understanding Check:

Find the x– and y–intercept of :

a. $3x - 2y = 18$

$3(0)-2y=18$ →

$3x-2(0)=18$ →

x	y
0	-9
6	0

b. $4x - 6y = -12$

$4(0)-6y=-12$ →

$4x-6(0)=-12$ →

x	y
0	2
-3	0

Shortcut (The cover up method):

Cover up the variable and coefficient that is being substituted with your finger to show that part "zeroing out". Solve the remaining equation.

Use the cover-up method to find the x– and y–intercept of :

Watch out for tricks!

a. $5x - 3y = -30$

x	y
0	10
-6	0

b. $-2x - 7y = 28$

x	y
0	-4
-14	0

c. $-3x + 9y = -18$

x	y
0	-2
6	0

d. $2x + 6y \cancel{- 24} = 0$ ↳ -24

x	y
0	-4
-12	0

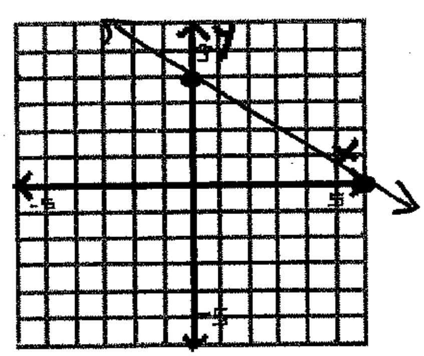
Example 2: Graphing Lines Using Intercepts

Graph $2x + 3y = 12$

Step 1: Find and plot the y-intercept.

Step 2: Find and plot the x-intercept.

x	y
0	4
6	0

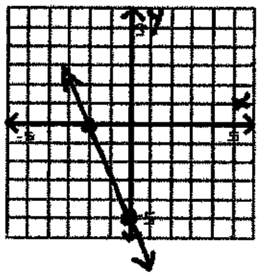


Step 3: Draw the line.

√ Understanding Check:
Graph using the x– and y–intercepts.

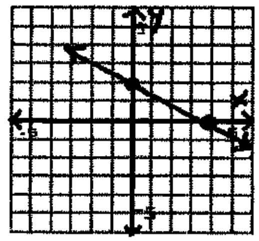
a. $5x + 2y = -10$

x	y
0	-5
-2	0



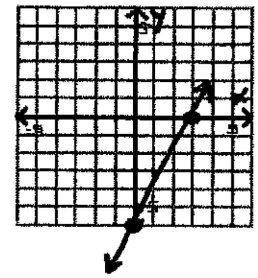
b. $-2x - 4y = -8$

x	y
0	2
4	0



c. $16x - 8y = 48$

x	y
0	-6
3	0



Activity by a 150 lb Person	Calories Burned per Minute
Bowling	3
Walking	5
Jogging	7
Bicycling	10
Swimming	11
Running	15

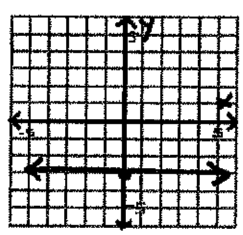
Example 4: Application: Why would a line ever be in standard form?
Use the table to write an equation in standard form to find the number of minutes a 150 lb person would need to bicycle and swim laps in order to burn 300 calories.

Define:
Let x = bicycling (min)
Let y = swimming laps (min)

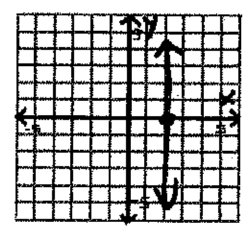
Write:
$10x + 11y = 300$

Example 3: Graphing Horizontal and Vertical Lines

a. Graph $y = -3$



b. Graph $x = 2$

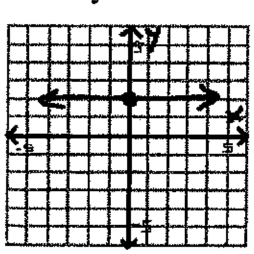


All equations with only a y variable will graph as a horizontal line.

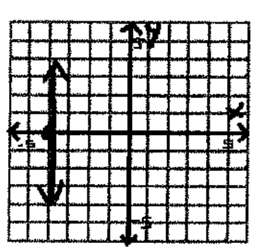
All equations with only an x variable will graph as a vertical line.

√ Understanding Check:
Graph each equation.

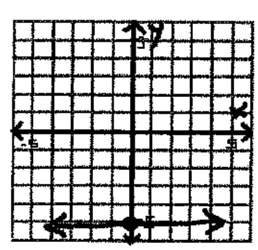
a. $y = 2$



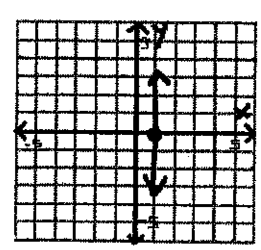
b. $x = -4$



c. $y = -5$



d. $x = 1$



5-4 Point-Slope Form (page 252)

Video Tutor Code: bae-0504

Point-Slope Form of a Linear Equation

You know the formula for slope is: $\cancel{(x_2-x_1)}\,\dfrac{y_2-y_1}{\cancel{x_2-x_1}} = m(x_2-x_1)$

If you multiply both sides of the equation by $(x_2 - x_1)$ and then remove the point (2) labels, you get the point-slope form of an equation:

$$= y_{\cancel{2}} - y_1 = m(x_{\cancel{2}} - x_1)$$

$$\boxed{y - y_1 = m(x - x_1)}$$

Example 1: Writing an Equation in Point-Slope Form (given the slope and one point)

* Highlighters really help students see the substitution.

1. Write an equation in point-slope form that has slope, -3, that passes through the point (-1, 7). Then change the equation to slope-intercept form (y=mx+b).

$\quad\;\; x_1\;\, y_1$

Step 1: Substitute -3 for m, and (-1, 7) for (x_1, y_1)

Step 2: Distribute -3

Step 3: Add 7 to both sides.

$$y - y_1 = m(x - x_1)$$
$$y - 7 = -3(x - -1)$$
$$y - 7 = -3(x + 1)$$
$$y - 7 = -3x - 3$$
$$\quad\quad +7$$
$$y = -3x + 4$$

2. Write an equation in point-slope form that has slope, $\frac{2}{3}$, that passes through the point (-6, 2). Then change the equation to slope-intercept form (y=mx+b).

Step 1: Substitute $\frac{2}{3}$ for m and (-6, 2) for (x_1, y_1)

Step 2: Distribute $2/3$

Step 3: Add 2 to both sides.

$$y - y_1 = m(x - x_1)$$
$$y - 2 = \frac{2}{3}(x - -6)$$
$$y - 2 = \frac{2}{3}(x + 6)$$
$$y - 2 = \frac{2}{3}x + 4$$
$$\quad\quad +2$$
$$y = 2/3\,x + 6$$

√ Understanding Check:

Write an equation in point-slope form that has the given slope, and passes through the given point. Then change each to slope-intercept form of the line.

1. slope: 2, passing through (10, 8)

x_1, y_1

$y - 8 = 2(x - 10)$

$y - 8 = 2x - 20$

$+8$

$y = 2x - 12$

2. slope: 4, passing through (-3, -8)

x_1, y_1

$y - {}^-8 = 4(x - {}^-3)$

$y + 8 = 4(x + 3)$

$y + 8 = 4x + 12$

-8

$y = 4x + 4$

3. slope: -2, passing through (2, -3)

x_1, y_1

$y - {}^-3 = -2(x - 2)$

$y + 3 = -2x + 4$

-3

$y = -2x + 1$

4. slope: -1, passing through (-3, -5)

x_1, y_1

$y - {}^-5 = -1(x - {}^-3)$

$y + 5 = -1(x + 3)$

$y + 5 = -x - 3$

-5

$y = -x - 8$

7. slope: $\frac{1}{2}$, passing through (-4, 5)

x_1, y_1

$y - 5 = \frac{1}{2}(x - {}^-4)$

$y - 5 = \frac{1}{2}(x + 4)$

$y - 5 = \frac{1}{2}x + 2$

$+5$

$y = \frac{1}{2}x + 7$

8. slope: $\frac{2}{3}$, passing through (6, - 9)

x_1, y_1

$y - {}^-9 = \frac{2}{3}(x - 6)$

$y + 9 = \frac{2}{3}x - 4$

-9

$y = \frac{2}{3}x - 13$

Example 2: Using Two Points to Write an Equation (finding the slope first)

Write an equation in point-slope form that passes through the points (6, -4) and (-3, 5). Then change the equation to slope-intercept form (y=mx+b).

Step 1: Find the slope (m) → $\frac{y_2-y_1}{x_2-x_1}$ → $m = \frac{5--4}{-3-6} = \frac{9}{-9} = -1$

Step 2: Chose just one point to be (x_1, y_1) → $(6, -4)$ $\boxed{m=-1}$

Step 3: Substitute m, x_1, and y_1 in the point-slope formula.

Step 4: Solve for y.

$$y - -4 = -1(x-6)$$
$$y+4 = -x+6$$
$$-4$$
$$y = -x+2$$

√ Understanding Check:

1. Find the slope-intercept form of a linear equation that passes through the points: (-1, 5) and (1, 9)

$m = \frac{9-5}{1--1} = \frac{4}{2} = 2$ $\boxed{m=2}$ $\boxed{(1,9)}$

$$y-9 = 2(x-1)$$
$$y-9 = 2x-2$$
$$+9$$
$$y = 2x+7$$

2. Find the slope-intercept form of a linear equation that passes through the points (1, 1) and (2, -2)

$m = \frac{-2-1}{2-1} = \frac{-3}{1} = -3$ $\boxed{m=-3}$ $\boxed{(1,1)}$

$$y-1 = -3(x-1)$$
$$y-1 = -3x+3$$
$$+1$$
$$y = -3x+4$$

3. Find the slope-intercept form of a linear equation that passes through the points (4, -1) and (-4, -3)

$m = \frac{-3--1}{-4-4} = \frac{-2}{-8} = \frac{1}{4}$ $\boxed{m=\frac{1}{4}}$ $\boxed{(4,-1)}$

$$y - -1 = \frac{1}{4}(x-4)$$
$$y+1 = \frac{1}{4}x-1$$
$$-1$$
$$y = \frac{1}{4}x-2$$

4. Find the slope-intercept form of a linear equation that passes through the points (-3, 3) and (1, -5)

$m = \frac{-5-3}{1--3} = \frac{-8}{4} = -2$ $\boxed{m=-2}$ $\boxed{(-3,3)}$

$$y-3 = -2(x--3)$$
$$y-3 = -2(x+3)$$
$$y-3 = -2x-6$$
$$+3$$
$$y = -2x-3$$

Example 3: Writing an Equation Using a Table or Word Description

If a table models a linear equation, you can use two points from the table to make an equation for the table.

$(2,-3)$ $(5,-9)$
$x_1\ y_1$ $x_2\ y_2$

1.

x	y
-4	9
2	-3
5	-9
9	-17

Step 1: Choose 2 points for (x_1, y_1) and (x_2, y_2)

Step 2: Find the slope (slope formula)

Step 3: use point-slope formula with (x_1, y_1)

Step 4: Solve for y.

$m = \frac{-9 - -3}{5-2} = \frac{-6}{3} = -2$

$\boxed{m=-2}$

$y - -3 = -2(x-2)$

$y + 3 = -2x + 4$

-3

$y = -2x + 1$

2. John noticed that when he ordered pizza on Monday, he got a 3-topping pizza for \$10.00. On Wednesday, he got a 1-topping pizza for \$8.00. If the price of the pizza follows a linear model, find the equation that best describes the relationship between the price of the pizza (p) and the number of toppings (t).

$(3, 10)$ and $(1, 8)$

$m = \frac{8-10}{1-3} = \frac{-2}{-2} = 1$

$\boxed{m=1}$

$y - 10 = 1(x-3)$

$y - 10 = x - 3$

$+10$

$\boxed{y = x + 7}$

√ Understanding Check:

Find the equation of the line for each table of values.

a.

x	y
-1	-5
3	7
-2	-8
2	4

$(3,7)$ $(2,4)$

$m = \frac{4-7}{2-3} = \frac{-3}{-1} = 3$

$\boxed{m=3}$

$y - 7 = 3(x-3)$

$y - 7 = 3x - 9$

$+7$

$y = 3x - 2$

b.

x	y
-2	4
4	1
8	-1
2	2

$(4,1)$ $(2,2)$

$m = \frac{2-1}{2-4} = \frac{1}{-2} = -\frac{1}{2}$

$y - 1 = -\frac{1}{2}(x-4)$

$y - 1 = -\frac{1}{2}x + 2$

$+1$

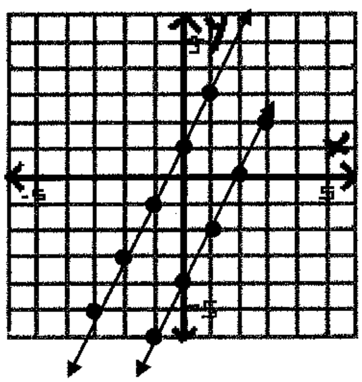
$y = -\frac{1}{2}x + 3$

Summary of Linear Equations

Slope-Intercept Form	Standard Form	Point-Slope Form
$y = mx + b$	$Ax + By = C$	$(y - y_1) = m(x - x_1)$
m is the slope and b is y – intercept.	A and B are not both 0.	(x_1, y_1) lies on the graph of the equation, and m is the slope.
$y = 3x + 5$	$2x + 3y = 6$	$y - 4 = 7(x - 3)$

5-5 Parallel and Perpendicular Lines (page 259) Video Tutor Code: bae-0505

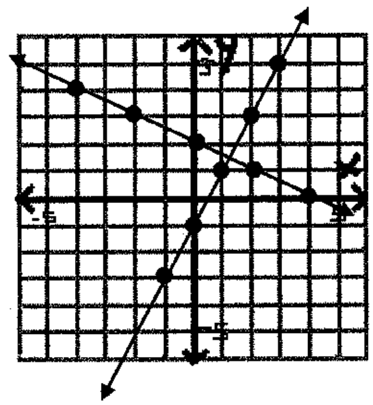
In the graph below, the lines are parallel. Write the equation of each line and compare their slopes.



$y = 2x + 1$

$y = 2x - 4$

In the graph below, the lines are perpendicular. Write the equation of each line and compare their slopes.



$y = 2x - 1$

$y = -\frac{1}{2}x + 2$

Slopes of Parallel Lines: slopes are the same

Slopes of Perpendicular Lines: slopes are reciprocal and opposite signs.

√ Understanding Check:

Circle the line that is **parallel** to the given line:

a. $y = 3x + 5$ — a. $y = 3x - 2$ b. $y = -\frac{1}{3}x - 1$ c. $y = -3x + 5$ d. $y = \frac{1}{3}x + 5$

b. $y = \frac{1}{2}x + 4$ — a. $y = 2x - 4$ b. $y = -\frac{1}{2}x - 1$ c. $y = -2x + 4$ d. $y = \frac{1}{2}x + 5$

Circle the line that is **perpendicular** to the given line:

a. $y = 3x + 5$ — a. $y = 3x - 2$ b. $y = -\frac{1}{3}x - 1$ c. $y = -3x + 5$ d. $y = \frac{1}{3}x + 5$

b. $y = -5x + 5$ — a. $y = -5x - 2$ b. $y = -\frac{1}{5}x - 1$ c. $y = 5x + 1$ d. $y = \frac{1}{5}x + 3$

Example 1: Determining Whether Lines Are Parallel

To determine if two lines are parallel from their equations, get both equations into the slope intercept form, then compare their slopes.

Are the graphs of $y = -\frac{1}{3}x + 5$ and $2x + 6y = 12$ parallel?

$m = \frac{1}{3}$

$$\begin{aligned} 2x + 6y &= 12 \\ -2x & \\ \hline \frac{6y}{6} &= \frac{-2x}{6} + \frac{12}{6} \\ y &= -\frac{1}{3}x + 2 \end{aligned}$$

$m = \frac{1}{3}$

Yes, the lines are parallel

√ Understanding Check:

Decide if each set of lines are parallel or not.

a. $y = -\frac{1}{2}x + 6$ and $2x - 4y = 12$

$m = -\frac{1}{2}$

$$\begin{aligned} 2x - 4y &= 12 \\ -2x & \\ \hline \frac{-4y}{-4} &= \frac{-2x}{-4} + \frac{12}{-4} \\ y &= \frac{1}{2}x - 3 \end{aligned}$$

$m = \frac{1}{2}$

*No, not parallel!

b. $-6x + 8y = -24$ and $y = \frac{3}{4}x - 7$

$$\begin{aligned} -6x + 8y &= -24 \\ +6x & \\ \hline \frac{8y}{8} &= \frac{6x}{8} - \frac{24}{8} \\ y &= \frac{3}{4}x - 3 \end{aligned}$$

$m = 3/4$

$m = 3/4$

*yes, parallel!

Example 2: Writing the Equations of Parallel Lines:

Write an equation for the line that contains (5, 1) and is parallel to $y = \frac{3}{5}x - 4$

x, y.

Step 1 Use the same slope for m.

Step 2 Substitute the point-slope formula.

Step 3 Solve for y.

$\boxed{m = \frac{3}{5}}$

$$\begin{aligned} y - 1 &= \frac{3}{5}(x - 5) \\ y - 1 &= \frac{3}{5}x - 3 \\ &\quad +1 \\ \hline y &= \frac{3}{5}x - 2 \end{aligned}$$

√ Understanding Check:

1. Write an equation for the line that contains (2, –6) and is parallel to

$y = 3x + 9$

$m = 3$

$$\begin{aligned} y - -6 &= 3(x - 2) \\ y + 6 &= 3x - 6 \\ &\quad -6 \\ \hline y &= 3x - 12 \end{aligned}$$

2. Write an equation for the line that contains (4, 3) and is parallel to

$y = \frac{1}{2}x - 4$

$\boxed{m = \frac{1}{2}}$

$$\begin{aligned} y - 3 &= \frac{1}{2}(x - 4) \\ y - 3 &= \frac{1}{2}x - 2 \\ &\quad +3 \\ \hline y &= \frac{1}{2}x + 1 \end{aligned}$$

Example 3: Determining Whether Lines Are Perpendicular

To determine if two lines are perpendicular from their equations, get both equations into the slope intercept form, then compare their slopes.

Are the graphs of $y = \frac{1}{3}x + 4$ and $3x + y = 7$ perpendicular? ★ Yes, perpendicular

$m = 1/3$

$-3x$

$y = -3x + 7$

$m = -3$

★ Reciprocal & opposite!

√ Understanding Check:

Decide if each set of lines are perpendicular or not.

a. $-6x + 8y = -24$ and $y = \frac{4}{3}x - 7$

$+6x$

$\frac{8y}{8} = \frac{6x}{8} - \frac{24}{8}$

$y = 3/4x - 3$ $m = 3/4$

$m = 4/3$

★ No, not opposite signs!

b. $y = 2x + 3$ and $-6y - 3x = 12$

$m = 2$

$+3x$

$\frac{-6y}{-6} = \frac{3x}{-6} + \frac{12}{-6}$

$y = -\frac{1}{2}x - 2$

$m = -\frac{1}{2}$

★ yes, opposite & reciprocal

Example 4: Writing Equations for Perpendicular Lines (not in the book)

Write an equation for the line that contains (4, 2) and is perpendicular to $y = -\frac{1}{3}x + 2$ $m = -\frac{1}{3}$

$m = 3$

Step 1 Change the given slope into its opposite sign and its reciprocal

Step 2 Use the point-slope formula.

Step 3 Solve for y.

$y - 2 = 3(x - 4)$

$y - 2 = 3x - 12$

$+2$

$y = 3x - 10$

√ Understanding Check:

1. Write an equation for the line that contains (4, –6) and is perpendicular to $y = 2x + 3$ $m = 2$

$m = -\frac{1}{2}$

$y - -6 = -\frac{1}{2}(x - 4)$

$y + 6 = -\frac{1}{2}x + 2$

-6

$y = -\frac{1}{2}x - 4$

2. Write an equation for the line that contains (–4, 7) and is perpendicular to $y = -\frac{2}{5}x - 2$ $m = -\frac{2}{5}$

$m = \frac{5}{2}$

$y - 7 = \frac{5}{2}(x - -4)$

$y - 7 = \frac{5}{2}(x + 4)$

$y - 7 = \frac{5}{2}x + 10$

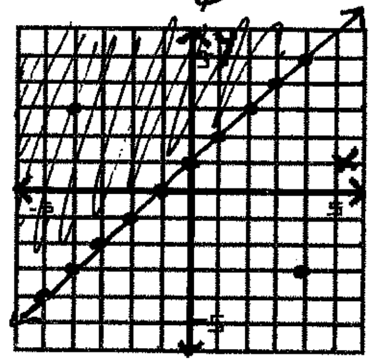
$+7$

$y = \frac{5}{2}x + 17$

5–6 Linear Inequalities

Investigating Linear Inequalities:

Shade in step #5 below.



1. Graph y = x + 1 on the coordinate plane.

2. Change the equals sign in the equation to a "greater than or equal to" symbol.

$y \geq x+1$

3. Choose three points from the graph that lie above the line. Substitute each one into the inequality to check if it makes a true statement.

* Answers will vary depending on what your students pick!

(,)   Example (-4, 3)   (,)

$3 \geq -4+1$

$3 \geq -3$

True

* All of these will be true!

4. Choose three points from the graph that lie below the line. Substitute each one into the inequality to check if it makes a true statement.

Example (4, -3)   (,)   (,)

$-3 \geq 4+1$

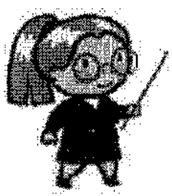
$-3 \geq 5$

Not true

* All of these will not be true!

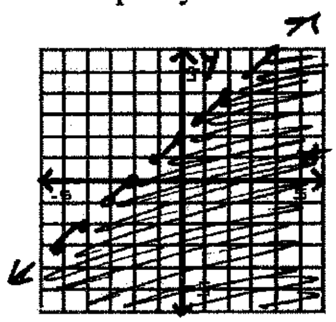
5. Shade the side of the line that contains all the points that will make the inequality true.

6. What if the inequality symbol was reversed? Which way would you shade then? Below the line or down.



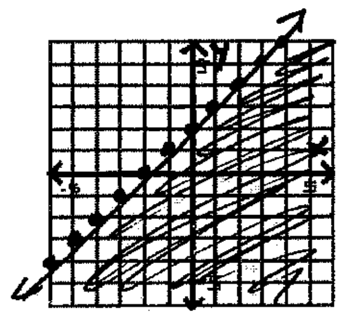
Graph < and > with a dashed line.

Example: $y < x + 2$



Graph ≤ and ≥ with a solid line.

Example: $y \le x + 2$



Test (0,0) for "true-ness".

$0 \le 0 + 2$

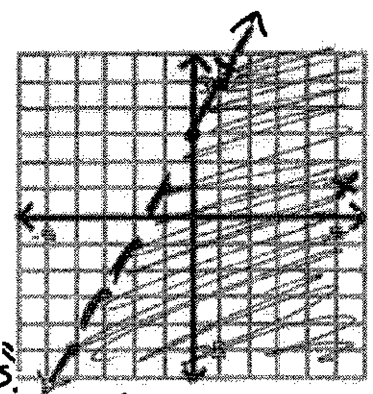
$0 \le 2$

True!

Example 1: Graphing an Inequality

dashed.

Graph: $y < 2x + 3$



Step 1: Plot points according to the y-intercept and slope.

Step 2: Draw in a dashed line.

Step 3: Test a point for "true-ness". and shade the true side → (0,0) works well.

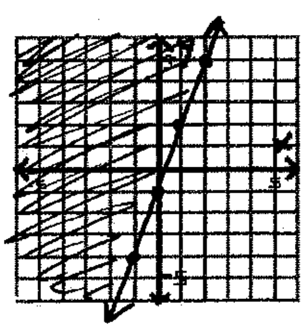
$0 < 2(0) + 3$

$0 < 3$ True!

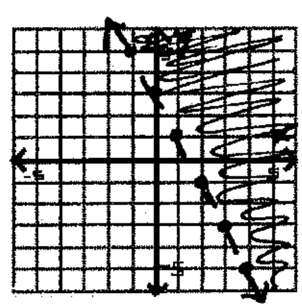
√ Understanding Check:

Graph each inequality:

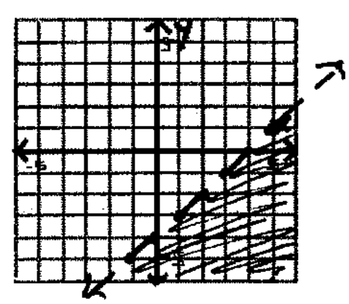
a. $y \ge 3x - 1$



b. $y > -2x + 3$



c. $y < x - 4$



Shortcut for shading:

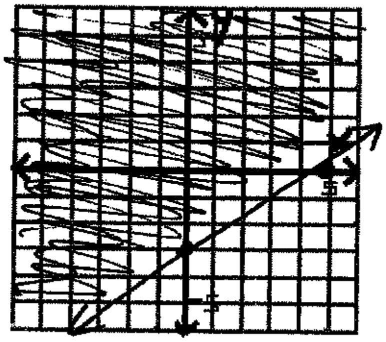
$y >$ "greater than" will shade above the line.

$y <$ "less than" will shade below the line.

Example 2: Graphing from Standard Form

Graph: $3x - 5y \le 15$

↑ solid



Step 1: Find and plot the intercepts

* use the cover-up shortcut. →

x	y
0	-3
5	0

Step 2: Draw a solid line.

Step 3: Test a point for "true-ness" & shade the true side.

(0, 0) → $3(0) - 5(0) \le 15$

$0 \le 15$

True!

* Or you can solve for y instead…but be careful!

$$3x - 5y \le 15$$
$$-3x$$
$$\frac{-5y}{-5} \le \frac{-3x}{-5} \frac{+15}{-5}$$

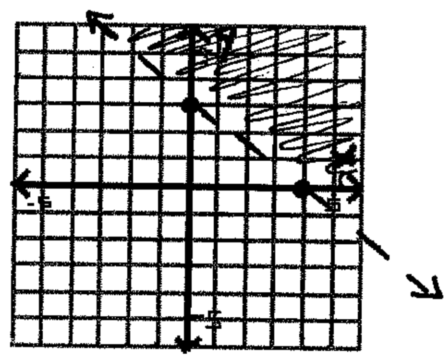
Flip!

$$y \ge \frac{3}{5}x - 3$$

√ Understanding Check:

a. Graph: $6x + 8y > 24$

Students can show work here… method will vary.



b. Graph: $4x - 5y \ge 10$

Students can show work here… method will vary.