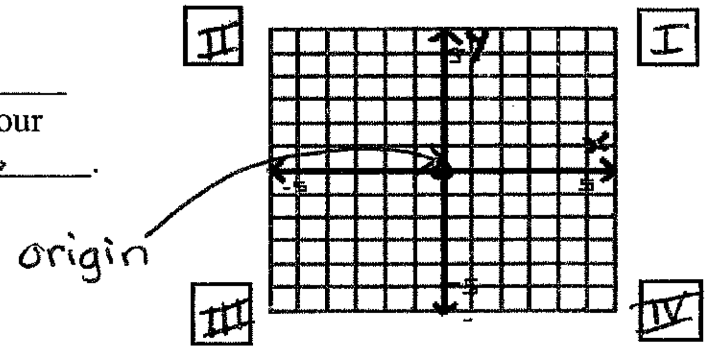
Graphing on the Coordinate Plane

Two number lines that intersect at right angles form a __coordinate__ __plane__. The horizontal axis is the __x-axis__ and the vertical axis is the __y-axis__.

The axes intersect at the __origin__ and divide the coordinate plane into four sections called __quadrants__.



Example 1: Graphing Points:
An ordered pair of numbers identifies the location of a point on the graph. These numbers are the __coordinates__ of a point on the graph.

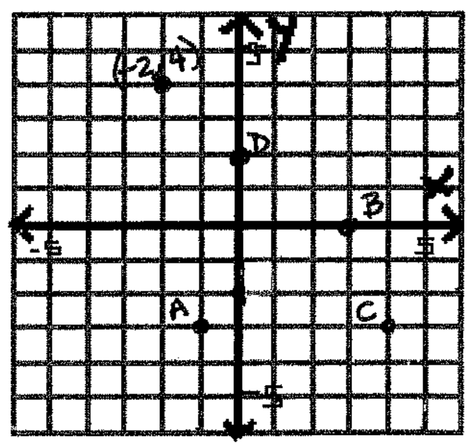
x y
Plot the ordered pair: (–2,4)

The x–coordinate tells you how far to move to the __left__ or __right__,

The y–coordinate tells you how far to move __up__ or __down__.

Let's plot some more points:

A (-1, -3) B (3, 0) C (4, -3) D (0, 2)



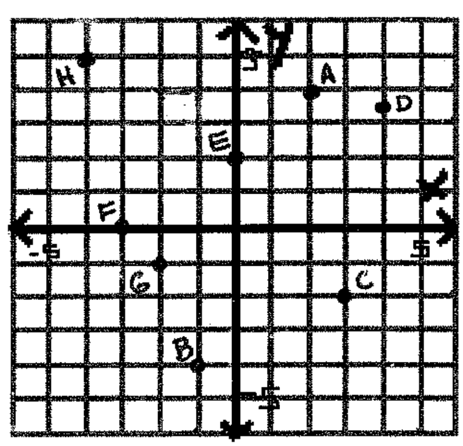
√ Understanding Check:
Graph the points on the coordinate plane.

A (2, 4) B (–1, –4)

C (3, –2) D (4, 3½)

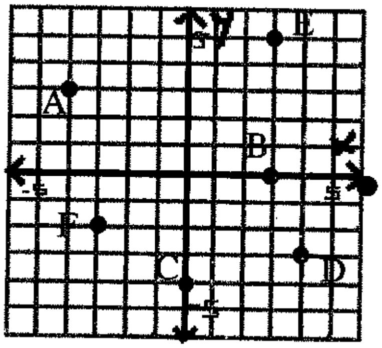
E (0, 2) F (-3, 0)

G (-2, -1) H (-4,5)



Example 2: Identifying Coordinates

Name the coordinates of the points in the graph below?



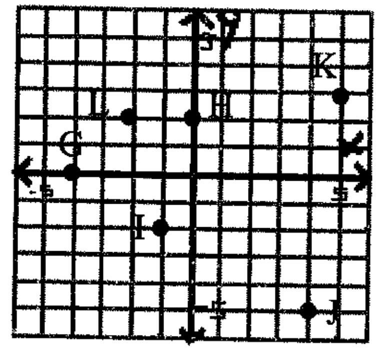
A $(-4,3)$ B $(3,0)$

C $(0,-4)$ D $(4,-3)$

E $(3,5)$ F $(-3,-2)$

√ Understanding Check:

Name the coordinates of each point in the graph below.



G $(-4,0)$ H $(0,2)$

I $(-1,-2)$ J $(4,-5)$

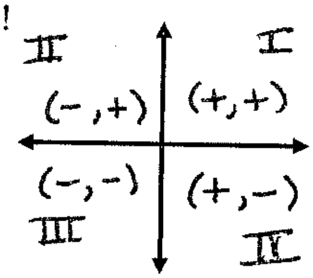
K $(5,3)$ L $(-2,2)$

Example 3: Predicting the Quadrant From the Signs

In which quadrant is each point located? Use Roman Numerals!

a. (5, 8) b. (-7, 10) c. (14, -2) d. (-100, -30)

I II IV III



√ Understanding Check:

In which quadrant is each point located? Use Roman Numerals!

1. (-3, 6) II
2. (-7, -4) III
3. (8, 12) I
4. (1, -4) IV
5. (2, -4) IV
6. (7, 3) I
7. (-1, 2) II
8. (-6, -5) III

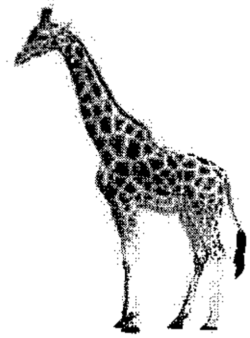
Relations and Functions

A relation is a set of ordered pairs. A relation can be shown in a variety of ways. Below a relation is shown as a table, list, graph, and mapping diagram.

Giraffe Heights

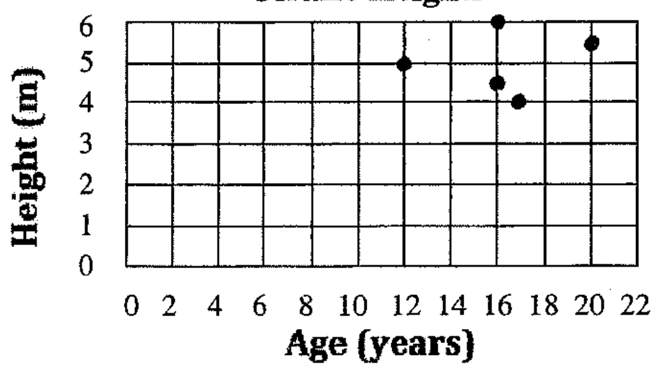
Age (years)	17	16	20	12	16
Height (meters)	4.0	4.5	5.5	5.0	6.0

(17, 4.0)
(16, 4.5)
(20, 5.5)
(12, 5.0)
(16, 6.0)

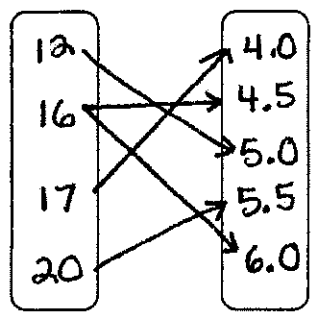


* The **domain** of a relation is the set of the x-values.
The **range** of relation is the set of the y-values.

Giraffe Heights



domain **range**

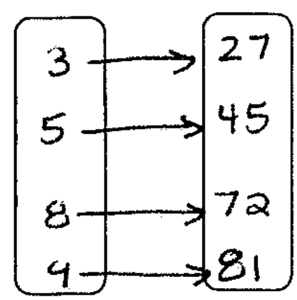


A relation that has only one value in the **range** for each x-value in the **domain** is called a function. Is the relation above a function? No
Why or why not? There are two values in the range for the value 16 in the domain.

Example 1: Identifying a Function Given a Table

Find the domain and range of the relation.
Is the relation a function?

domain **range**



Earning Money

Hours of Work	3	5	8	9
Amount Earned	$27	$45	$72	$81

Domain: {3, 5, 8, 9}

Range: {27, 45, 72, 81}

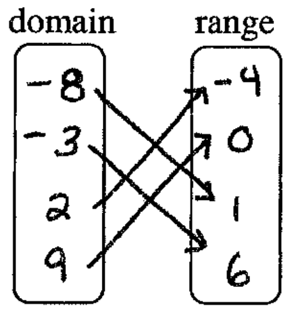
Is this relation a function? Yes

√ Understanding Check:

Find the domain and range of each relation. Is each relation a function?

a.

x	y
-3	6
2	-4
9	0
-8	1

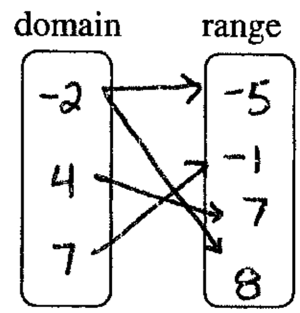


function? Yes

b.

x	y
-2	-5
4	7
-2	8
7	-1

If the x-values repeat, it's not a function.

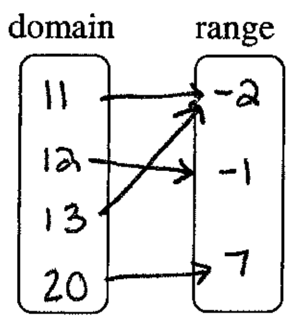


function? No

Example 2: Using a List

Find and list the domain and range of each relation.

a. {(11, –2), (12, –1), (13, –2), (20, 7)}



Is the relation a funtion? Yes

b. {(–2, –1), (–1, 0), (6, 3), (–2, 1)}

Domain: {-2, -1, 6}
Range: {-1, 0, 1, 3}

Is the relation a funtion? No

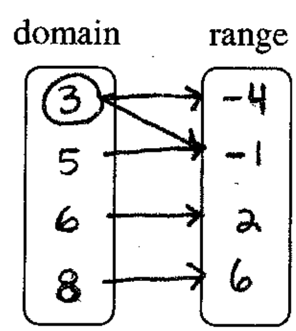
√ Understanding Check

a. {(3, –2), (8, 1), (9, 2), (5, 3)}

Domain: {3, 5, 8, 9}
Range: {-2, 1, 2, 3}

Do any of the domain values repeat? No
Is the relation a funtion? Yes

b. {(5, -1),(3, –1),(6, 2),(8, 6),(3, –1)}



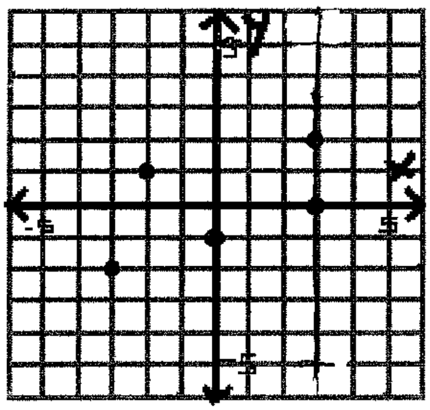
Is the relation a funtion? No

Example 3: Using the Vertical–Line Test

Find the domain and range of each relation. Use the vertical–line test to determine whether each relation is a function.

a. {(3, 0), (–2, 1), (0, –1), (–3, –2), (3, 2)}

Step 1: Graph the ordered pairs on a coordinate plane.



Step 2: Pass a vertical pencil across the graph.

Does the pencil pass through two points at the same time?

Which two points? (3,0) and (3,2)

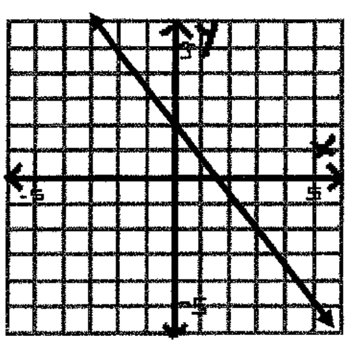
A function will NOT have two points in a vertical line.

This graph Not a function.

√ Understanding Check:

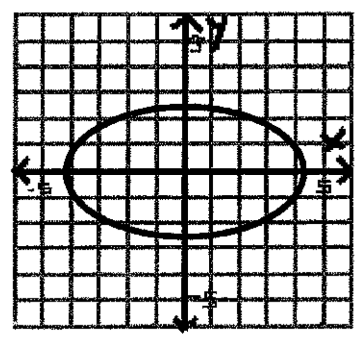
Decide if each of the following graphs are functions.

a.



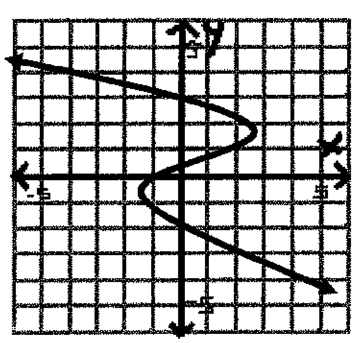
yes

b.



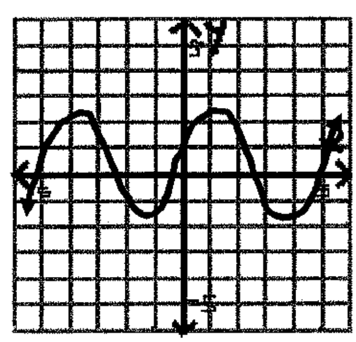
No

c.



No

d.

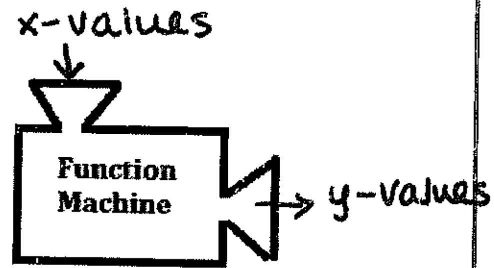


yes

Function Rules, Tables, and Graphs

A function rule is an equation that describes a function. You can think of a function rule as an input–output machine.

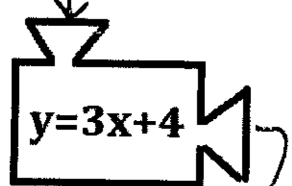
The x-values are the set of input values.
The y-values are the set of output values.



Example 1: Evaluating a Function Rule

If you know the input values, you can use a function rule to find the output values. The output values depend on the input values.

x	y
-3	-5
-2	-2
-1	1
0	4
1	7
2	10
3	13



* I use this area to put in one value at a time, erase, and do it again with the next value.

An equation could also be written with y as f(x).
So y = –2x + 1 could be written as $f(x) = -2x+1$.
This is known as function notation.

Don't let this scare you. You can even cross it out and make it a "y" if you want.

a. Evaluate: $f(x) = -2x + 1$ for the domain $\{-2 \le x \le 2\}$

Show work here:

$f(x) = -2(-2)+1$
$4+1$
5

$f(x) = -2(-1)+1$
$2+1$
3

$f(x) = -2(0)+1$
$0+1$
1

$f(x) = -2(1)+1$
$-2+1$
-1

$f(x) = -2(2)+1$
$-4+1$
-3

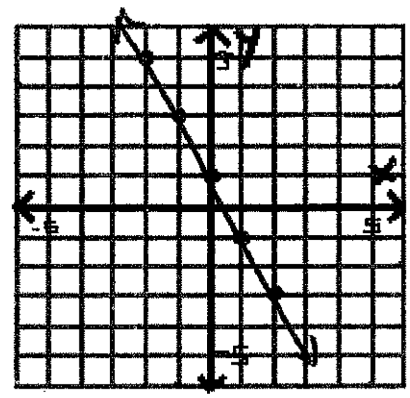
x	f(x)
-2	5
-1	3
0	1
1	-1
2	-3

An interesting thing happens when we graph the input/output pairs as coordinate points on a graph. Use the values from the table you just made to make five sets of coordinate pairs and plot them on the graph.

(-2 , 5) (1 , -1)

(-1 , 3) (2 , -3)

(0 , 1)



What do you notice about the points? They all line up.

What do you think would happen if we chose more x-values to evaluate with the same function rule? They would be in line too!

For this reason, we connect all the points with a line with arrows on the end.

*Draw the line after answering the questions.

√ **Understanding Check:**

Find the range of each function for the domain, $\{-2 \le x \le 2\}$. Make a table for each set of values. Then graph the coordinate pairs. Connect the points with a line.

a. $f(x) = 2x - 1$

$f(x) = 2(-2) - 1$
$-4 - 1$
-5

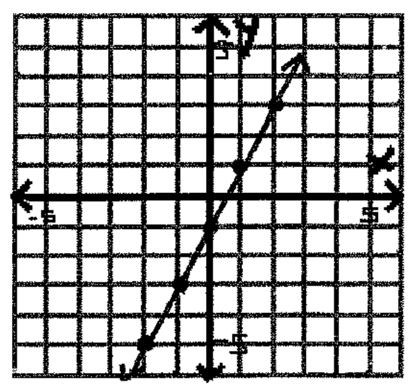
$2(-1) - 1$
$-2 - 1$
-3

$2(0) - 1$
$0 - 1$
-1

$2(1) - 1$
$2 - 1$
1

$2(2) - 1$
$4 - 1$
3

x	y
-2	-5
-1	-3
0	-1
1	1
2	3



b. $y = -x + 2$

$y = -(-2) + 2$
$2 + 2$
4

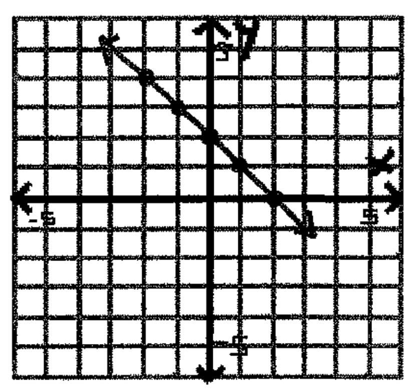
$-(-1) + 2$
$1 + 2$
3

$-(0) + 2$
$0 + 2$
2

$-(1) + 2$
$-1 + 2$
1

$-(2) + 2$
$-2 + 2$
0

x	y
-2	4
-1	3
0	2
1	1
2	0



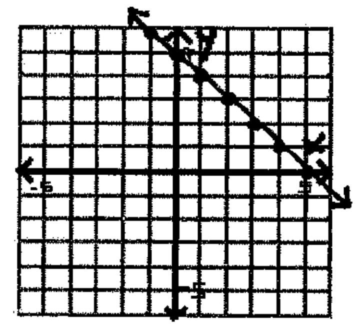
Each point on the graph of a line is a Solution that makes the equation true.

For example: If x + y = 5, how many solutions can you think of?

$1+4=5 \longrightarrow (1,4)$
$2+3=5 \longrightarrow (2,3)$
$3+2=5 \longrightarrow (3,2)$
$4+1=5 \longrightarrow (4,1)$
$0+5=5 \longrightarrow (0,5)$
$5+0=5 \longrightarrow (5,0)$
$-1+6=5 \longrightarrow (-1,6)$

* Let students suggest these!

Now graph some of your solutions.



Example 2: Verifying Points on a Line

To verify that a point lies on a line, substitute its coordinates in for x and y in the equation. If doing so gives a true statement, then the point is on the line.

a. Determine whether (3, –5) lies on the graph of y = –3x + 4

Step 1: Substitute 3 for x, -5 for y
Step 2: Simplify
Step 3: Compare left to right, (is it true?)

$-5 = -3(3)+4$
$-5 = -9+4$
$-5 = -5$ yes ✓ the point is on the line.

b. Determine whether (8, 4) lies on the graph of 3y = 2x – 1

Step 1: Substitute 8 for x, 4 for y
Step 2: Simplify
Step 3: Compare left to right (is it true?)

$3(4) = 2(8)-1$
$12 = 16-1$
$12 = 15$ No the point is not on the line.

√ Understanding Check:

a. Determine whether (1, 6) lies on the graph of y = 4x – 2

x y

$6 = 4(1)-2$
$6 = 4-2$
$6 = 2$ No

b. Determine which points lie on the graph of 3y + 5x = 4

a. (4, 3)
$3(3)+5(4)=4$
$9+20=4$
$29=4$
No

b. (5, -7)
$3(-7)+5(5)=4$
$-21+25=4$
$4=4$
Yes

c. (-2, 1)
$3(1)+5(-2)=4$
$3-10=4$
$-7=4$
No

d. (3, -1)
$3(-1)+5(3)=4$
$-3+15=4$
$12=4$
No

Writing a Function Rule

Example 1: Using a table and graph to model a function rule.

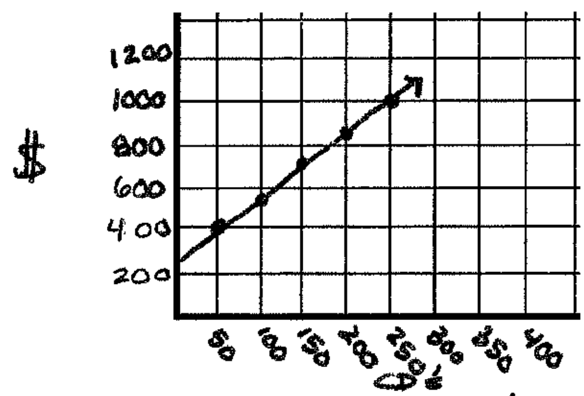
Suppose your group recorded a CD. Now you want to copy and sell it. One company charges \$250 for making a master CD and designing the art for the cover. There is also a cost of \$3 to burn each CD. The total cost P(c) depends on the number of CD's (c) burned.

Write a function rule to show the total cost of the CD's. P(c) = 3x + 250

Now create a table and graph to model the function rule.

*Guide students in choosing their x values → Ask them to think about how many CD's they need for their small fan base.

c	P(c)
50	400
100	550
150	700
200	850
250	1000



*This is a good time to start introducing Scaling. & labels.

What does the total cost depend on? the number of CD's you have made.

We call that the independant variable. What changed as the number of CD's increased? the cost/\$ We call that the dependant variable.

√ Understanding Check:

Suppose you need to deliver boxes of your new CD's to several cities. You find a company that charges \$30.00 to rent the truck and \$2.00 per mile.

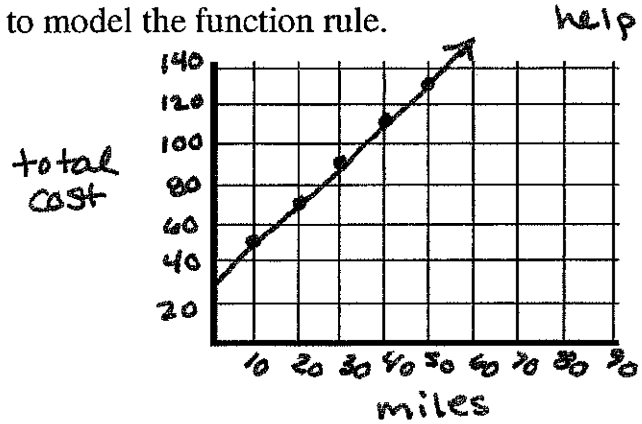
What does the total cost (c) depend on? the number of miles (m) you drive

Write a function rule to model the total cost: C = 2m + 30

Now create a table and graph to model the function rule. *Students will need help w/ scaling.

*Try to keep the students close to home ☺

m	c
10	50
20	70
30	90
40	110
50	130



*These are good problems to come back to when you introduce y-intercepts.

What was the independent variable in this problem? # of miles driven

What was the dependent variable in this problem? the total cost

Example 2: Finding the Function Rule

Can you find the function rule for a table of values? You can if you are good at finding patterns. Try to figure out how each output value is related to the input value.

Find the function rule for each table of values.

* At this point we are guessing & checking. I give them hints (look next to zero.)

1.

x	f(x)
-2	-4
-1	-2
0	0
1	2
2	4

$f(x) = 2x$

2.

x	y
-2	3
-1	4
0	5
1	6
2	7

$y = x + 5$

3.

x	f(x)
-2	-7
-1	-4
0	-1
1	2
2	5

$f(x) = 3x - 1$

√ Understanding Check:

a.

x	y
-2	-10
-1	-5
0	0
1	5
2	10

$y = 5x$

b.

x	f(x)
-2	-6
-1	-5
0	-4
1	-3
2	-2

$f(x) = x - 4$

c.

x	y
-2	1
-1	3
0	5
1	7
2	9

$y = 2x + 5$

Example 3: Writing a function rule for a situation.

a. The total distance (***d***) traveled after (***h***) hours at a constant speed of 20 mph.

$d = 20h$

b. The height of an object in feet (*f*), when you know the height in inches (*i*).

$f = \frac{i}{12}$

c. The profit (***p***) you make from mowing lawns (*l*) at \$10 a lawn, less the cost of purchasing the mower for \$100.

$p = 10l - 100$

√ Understanding Check:

1. The pay (***p***) a worker earns whose hourly wage is \$9.50 an hour (***h***).

$p = 9.50h$

2. The price (***p***) of purchasing a pizza for \$10 plus \$2 for each topping (***t***).

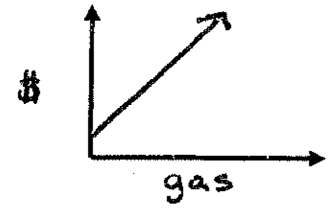
$p = 2t + 10$

Recognizing a Graph as a Pictoral Representation of a Function

Name the independent variable and the dependent variable for each relationship below. Then sketch a graph to represent each relationship described below.

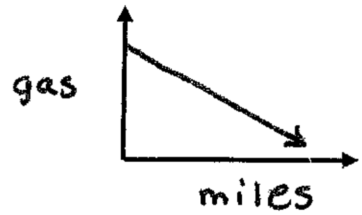
1. The amount of money you would pay for gasoline as you fill your car's tank from almost empty to full.

   independent variable: gas /# of gallons
   dependent variable: $



2. The amount of gasoline in the same car as you then drive it 200 miles.

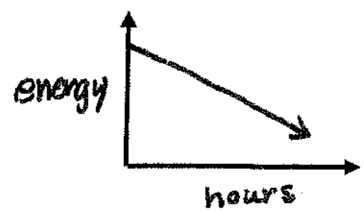
   independent variable: miles
   dependent variable: gas



√ Understanding Check:

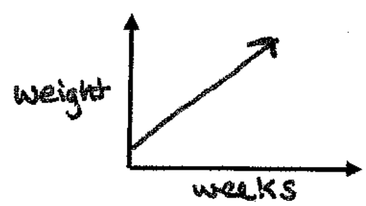
1. The amount of energy in a fully charged cell phone as someone makes a long four-hour phone call.

   independent variable: hours
   dependent variable: energy



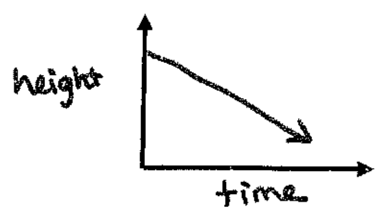
2. The weight of a puppy from birth to 12 weeks.

   independent variable: weeks
   dependent variable: weight



3. The height of a burning birthday candle over time.

   independent variable: time
   dependent variable: height

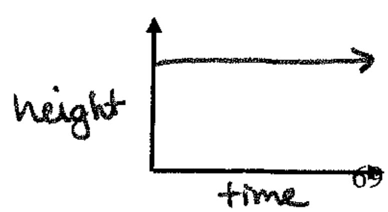


4. The amount of money in a savings account opened with 20 dollars that gets regular deposits of 10 a week.

   independent variable: weeks
   dependent variable: $ or savings

* 5. The height of a birthday candle unlit over time.

   independent variable: time
   dependent variable: height



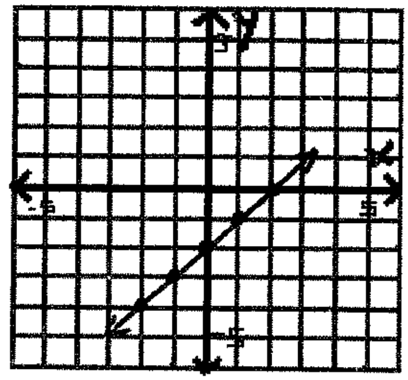
Exploring Different Types of Functions

Make a table of values and a graph for each function rule. Use $\{-2 \le x \le 2\}$ for the domain for each problem.

1. $y = x$

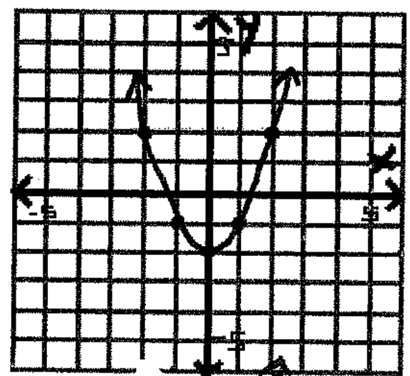
*Students can use this space to show work.

x	y
-2	-4
-1	-3
0	-2
1	-1
2	0



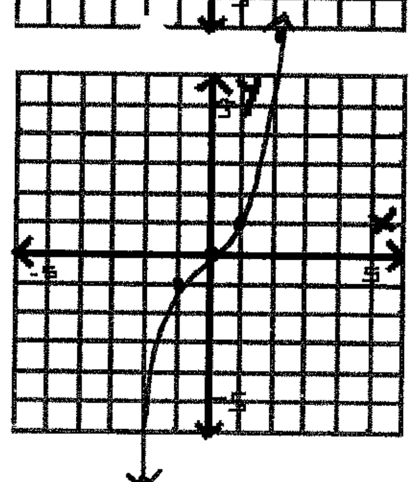
2. $y = x^2$

x	y
-2	2
-1	-1
0	-2
1	-1
2	2



3. $y = x^3$

x	y
-2	-8
-1	-1
0	0
1	1
2	8



4. $y = |x|$

x	y
-2	2
-1	1
0	0
1	1
2	2